Vip Formulas for Power and Harmonic Measurements

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Abstract– *The Vip uses a variety of algorithms to compute RMS voltage and current, real, reactive, and apparent power, true and displacement power factor, phase angle, total harmonic distortion, and harmonic magnitudes and phases. The formulas for these algorithms are detailed here.*

1 Introduction

The Vip samples four pairs of voltages and currents. From these samples it computes RMS voltage and current; real, reactive, and apparent power; power factor and displacement power factor, phase angle, voltage and current THD, and harmonic magnitudes and phases. The raw waveforms are sampled at a rate of 256 samples per powerline cycle (usually 60Hz). Here the complications of A/D quantization, scaling, finite precision math, gain and offset correction, hardware temperature drift compensation, harmonic magnitude and phase corrections, and synchronization with the powerline frequency are not discussed. Thus, assume all measurements are in volts or amperes, with infinite precision, and perfectly synchronized such that 256 samples is exactly one powerline cycle (hereafter called a 60 Hz cycle, though the actual frequency may be from 46 to 70 Hz). The formulas given here are not necessarily those peformed by the Vip, but are numerically equivalent expressions.

1.1 Notation and Sampled Data

The Vip samples four channels of voltage and four channels of current. Let $v_1[n], v_2[n], v_3[n], v_4[n]$ and $i_1[n], i_2[n], i_3[n], i_4[n]$ represent the sampled voltages and currents for the four channels. In a single 60Hz cycle, the samples are indexed in the range $0 \leq n \leq 255$. Where the channel number is not relevant, the subscript may be dropped. Where multiple cycles of data are needed, a superscript is added: $v_j^m[n]$ is the nth voltage sample for the jth channel for the mth cycle, where $0 \leq n \leq 255$, $1 \leq j \leq 4$, and $m > 0$.

2 Independent Channels/Single Phase

In this recording mode, each pair of voltage and current channels are used independently. Three phase wye and delta calculations are extensions to the formulas for the single phase case.

2.1 RMS Voltage and Current

The rms value is computed once per cycle for each channel of voltage and current. The voltage rms value is computed by

$$\mathrm{VRMS} = \sqrt{\frac{1}{256}\sum_{n=0}^{255}\big(v[n]\big)^2}\,. \tag{1}$$

Similarly, the current rms value is given by

$$\mathrm{IRMS} = \sqrt{\frac{1}{256}\sum_{n=0}^{255}\big(i[n]\big)^2}\,. \tag{2}$$

2.2 Real Power

Real power is computed once per cycle for each pair of voltage and current channels. The real power value is computed by

$$\mathrm{W} = \frac{1}{256}\sum_{n=0}^{255} v[n]i[n]. \tag{3}$$

Note that real power is signed to indicate direction of power flow.

2.3 Apparent Power

Apparent power is computed once per cycle for each pair of voltage and current channels. The apparent power value is computed by

$$\mathrm{VA} = \mathrm{VRMS} \times \mathrm{IRMS}\,. \tag{4}$$

2.4 Harmonics

An FFT of each voltage and current channel is computed every cycle. Since harmonics only to the 51st are required, the anti-aliased, sampled data is smoothed and downsampled by a factor of two before a 128-point FFT is performed. The smoothing is done by averaging each pair of data points. The complex FFT result, including the smoothing and downsampling, is given by

$$V[k] = \sum_{n=0}^{127} \frac{1}{2}(x[2n] + x[2n+1])e^{-j2\pi kn/128} \tag{5}$$

for $k = 0, \ldots, 63$. Here j represents $\sqrt{-1}$. Since the FFT is done on a single 60Hz cycle of data, the index k also represents the harmonic number. The 128 point FFT gives a decomposition into 64 harmonics of 60Hz. For specific channels and cycle numbers, the notation $V_j^m[k]$ and $I_j^m[k]$ denote the FFT value for jth channel, for the mth cycle number, for the kth harmonic. The real and imaginary parts of $V[k]$ are denoted by $V_x[k]$ and $V_y[k]$, respectively. The real and imaginary parts for channel j are $V_{jx}[k]$ and $V_{jy}[k]$.

The harmonic magnitudes and phases are computed once per second, to provide some averaging and to reduce transient effects. The one-cycle FFT values are averaged over the M cycles which comprise each second, to form

$$\overline{V[k]} = \frac{1}{M} \sum_{m=1}^{M} V^m[k]\,. \tag{6}$$

The kth harmonic magnitude is then given by

$$\mathrm{VMAG}[k] = \left|\overline{V[k]}\right| = \sqrt{\left(\overline{V_x[k]}\right)^2 + \left(\overline{V_y[k]}\right)^2}\,, \tag{7}$$

and the raw kth harmonic phase angle is

$$\mathrm{V}\theta[k] = \angle\overline{V[k]} = \arctan\left(\frac{\overline{V_y[k]}}{\overline{V_x[k]}}\right). \tag{8}$$

The arctan function is the four quadrant inverse tangent, with a range of -180 to $+180$ degrees. The current magnitudes and phase angles are computed in the same manner. The voltage harmonic phase angles are referred to the first voltage channel's first harmonic phase angle. The current harmonic phase angles are then referred to their cooresponding voltage 60Hz phase angles. This two-step algorithm proceeds as follows for the jth channel:

$$\begin{array}{lll} 1) & \mathrm{V}\theta_j[k] & = \mathrm{V}\theta_j[k] - k\mathrm{V}\theta_1[1], \quad k = 1, \ldots, 51 \\ 2) & \mathrm{I}\theta_j[k] & = \mathrm{I}\theta_j[k] - k\mathrm{V}\theta_j[1], \quad k = 1, \ldots, 51. \end{array}$$

2.5 Phase Angle

The phase angle, θ, is the angular phase shift between the 60Hz voltage and current sinusiods. It is computed every cycle, and is simply

$$\theta = \mathrm{I}\theta[1] - \mathrm{V}\theta[1], \tag{9}$$

where $\mathrm{I}\theta[1]$ and $\mathrm{V}\theta[1]$ are the phase angles for the 1st harmonic (60Hz). These phase angles are computed using (8) on the raw FFT outputs instead of the one second average, with $k = 1$.

2.6 Reactive Power

Reactive power is computed every cycle for each pair of voltage and current channels. The result is given by

$$\mathrm{VAR} = \sum_{k=1}^{51} \left(V_x[k]I_y[k] - I_x[k]V_y[k]\right)\,. \tag{10}$$

Each $V_x[k]I_y[k] - I_x[k]V_y[k]$ term is the reactive power contributed by harmonic k.

2.7 Power Factor

Power factor is computed once per cycle for each pair of voltage and current channels. The result is given by

$$\mathrm{PF} = \left|\frac{\mathrm{W}}{\mathrm{VA}}\right|\,, \quad \begin{cases} \text{no suffix,} & \text{for } \theta = 0 \text{ or } \theta = \pm 180 \\ \text{lead,} & \text{for } 0 < \theta < 180 \\ \text{lag,} & \text{for } -180 < \theta < 0 \end{cases} \tag{11}$$

This expression is also known as true power factor, since it includes the effects of harmonics.

2.8 Displacement Power Factor

Displacement power factor is computed once per cycle for each pair of voltage and current channels. This quantity represents only the 60Hz contribution to the true power factor. The result is computed by

$$\mathrm{dPF} = |\cos\theta|\,, \quad \begin{cases} \text{no suffix,} & \text{for } \theta = 0 \text{ or } \theta = \pm 180 \\ \text{lead,} & \text{for } 0 < \theta < 180 \\ \text{lag,} & \text{for } -180 < \theta < 0 \end{cases} \tag{12}$$

2.9 THD

Total harmonic distortion, computed every second for each channel of voltage and current, is given in percent by

$$\mathrm{VTHD}\frac{\sqrt{\displaystyle\sum_{k=2}^{51}(\mathrm{VMAG}[k])^2}}{\mathrm{VMAG}[1]} \times 100. \tag{13}$$

Since this THD definition is referred to the fundamental (as opposed to the RMS value), it may be over 100%.

3 Three Phase Wye

In a three phase wye hookup, each pair of voltage and current channels are handled in the same manner as the single phase hookup. The first three pairs are also grouped together to form total power quantities.

3.1 Total Powers

Total real, reactive, and apparent power are computed and displayed but not recorded in wye mode. The three phase totals are the sum of the individual phases:

$$\mathrm{W}_{TOT} = \mathrm{W}_1 + \mathrm{W}_2 + \mathrm{W}_3 \tag{14}$$

$$\mathrm{VAR}_{TOT} = \mathrm{VAR}_1 + \mathrm{VAR}_2 + \mathrm{VAR}_3 \tag{15}$$

$$\mathrm{VA}_{TOT} = \mathrm{VA}_1 + \mathrm{VA}_2 + \mathrm{VA}_3\,. \tag{16}$$

All these totals are computed every second from one second averages. The values are displayed on the front panel and then discarded.

3.2 Total Power Factors, Phase Angle

These total quantities are computed as weighted averages of the three phases, weighted by apparent power:

$$\mathrm{PF}_{TOT} = \frac{\mathrm{PF}_1\mathrm{VA}_1 + \mathrm{PF}_2\mathrm{VA}_2 + \mathrm{PF}_3\mathrm{VA}_3}{\mathrm{VA}_{TOT}} \tag{17}$$

$$\mathrm{dPF}_{TOT} = \frac{\mathrm{dPF}_1\mathrm{VA}_1 + \mathrm{dPF}_2\mathrm{VA}_2 + \mathrm{dPF}_3\mathrm{VA}_3}{\mathrm{VA}_{TOT}} \tag{18}$$

$$\theta_{TOT} = \frac{\theta_1\mathrm{VA}_1 + \theta_2\mathrm{VA}_2 + \theta_3\mathrm{VA}_3}{\mathrm{VA}_{TOT}} \tag{19}$$

All these totals are computed every second from one second averages. The values are displayed on the front panel and then discarded.

4 Three Wire Delta

With a three wire delta circuit, individual phase powers and power factors cannot be computed without imposing assumptions such as a balanced load, balanced source, etc. The Vip only computes total quantities in this mode. These values are computed and recorded as channel one data. As in the wye case, these values are computed once per cycle. The fourth channel is treated as an extra single phase channel with power calculations as detailed in Section 2. Real and reactive power are calculated using the two-wattmeter method, using voltage channels 1 and 2, and current channels 1 and 3. The Vip is connected as a delta, with each voltage channel connected from phase to phase.

4.1 Real Power

Real power is computed using the two-wattmeter method. This requires two voltage and current channels to compute the three phase total. Voltage channels one and two are used with current channels one and three:

$$\mathrm{W}_{TOT} = \frac{1}{256}\left(\sum_{n=0}^{255} v_1[n]i_1[n] - \sum_{n=0}^{255} v_2[n]i_3[n]\right). \tag{19}$$

4.2 Reactive Power

Reactive power is computed using the two-wattmeter method. This requires two voltage and current channels to compute the three phase total. Voltage channels one and two are used with current channels one and three:

$$\begin{aligned}\mathrm{VAR}_{TOT} = &\sum_{k=1}^{51}\left(V_{1x}[k]I_{1y}[k] - I_{1x}[k]V_{1y}[k]\right) \qquad (20)\\ &- \sum_{k=1}^{51}\left(V_{2x}[k]I_{3y}[k] - I_{3x}[k]V_{2y}[k]\right).\end{aligned}$$

4.3 Apparent Power

Apparent power is computed by:

$$\mathrm{VA}_{TOT} = \sqrt{(\mathrm{W}_{TOT})^2 + (\mathrm{VAR}_{TOT})^2}\,. \tag{21}$$

4.4 Phase Angle

The phase angle, θ, is the angular phase shift between the 60Hz voltage and current sinusiods. Since the actual phase current cannot be measured in a three wire delta hookup, the 60Hz component of the real and reactive powers must be used to compute a total three-phase phase angle. The 60Hz component of the reactive power, $\mathrm{VAR}_{TOT}[1]$ is computed using (20) with $k = 1$ (since 60Hz is the 1st harmonic), giving

$$\begin{aligned} \mathrm{VAR}_{TOT}[1] = V_{1x}[1]I_{1y}[1] - I_{1x}[1]V_{1y}[1] \\ - V_{2x}[1]I_{3y}[1] + I_{3x}[1]V_{2y}[1]. \end{aligned} \tag{22}$$

The 60Hz component of the real power, $\mathrm{W}_{TOT}[1]$, can be obtained in an analogous fashion using

$$\begin{aligned} \mathrm{W}_{TOT}[1] = V_{1x}[1]I_{1x}[1] + I_{1y}[1]V_{1y}[1] \\ - V_{2x}[1]I_{3x}[1] - I_{3y}[1]V_{2y}[1]. \end{aligned} \tag{23}$$

This results in the following expression for θ_{TOT}:

$$\theta_{TOT} = \arctan\left(\frac{\mathrm{VAR}_{TOT}[1]}{\mathrm{W}_{TOT}[1]}\right). \tag{24}$$

4.5 Power Factors

Power factor and displacement power factor are computed with (11) and (12), with the use of W_{TOT}, VA_{TOT}, and θ_{TOT} instead of the single phase W, VA, and θ.

5 Four Wire Delta

With a four wire delta circuit, individual phase powers and power factors cannot be computed without imposing assumptions such as a balanced load, balanced source, etc. The Vip only computes total quantities in this mode. These values are computed and recorded as channel one. These computations happen once per cycle, as in the wye case. The fourth channel is treated as an extra single phase channel with power calculations as detailed in Section 2. Real and reactive power are calculated using the three-wattmeter method, which uses all three voltage and current channels. The Vip itself is connected as a wye, with each voltage channel measuring from phase to neutral.

5.1 Total Powers

Real and reactive total power is computed as the sum of the individual channels' real and reactive powers, computed as if they were part of a wye circuit. Thus, (14) and (15) can be used, with (3) and (10) used to compute channel powers as in the wye case. Total apparent power is computed with (21).

5.2 Phase Angle

The phase angle is computed with (24). To compute the 60Hz real and reactive power used in (24), all three voltage and current channels are utilized, as per the three-wattmeter methodology. The expressions for $\mathrm{W}_{TOT}[1]$ and $\mathrm{VAR}_{TOT}[1]$ become

$$\mathrm{W}_{TOT}[1] = \sum_{j=1}^{3} V_{jx}[1]I_{jx}[1] + I_{jy}[1]V_{jy}[1] \tag{25}$$

and

$$\mathrm{VAR}_{TOT}[1] = \sum_{j=1}^{3} \left(V_{jx}[1]I_{jy}[1] - I_{jx}[1]V_{jy}[1]\right). \tag{26}$$